Can a marginally open universe amplify magnetic fields?

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Abstract. In a series of recent papers, including arXiv:1210.1183, it was claimed that large-scale magnetic fields generated during inflation in a spatially open universe could remain astrophysically significant at the present time since they experienced *superadiabatic amplification* specific to an open universe. We reexamine this assertion and show that, on the contrary, large-scale magnetic fields in a realistic open universe decay in much the same manner as they would in a spatially flat universe. Consequently, their amplitude today is extremely small ($B_0 \lesssim 10^{-59}$ G) and is unlikely to be of astrophysical significance.

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Contents

1 Introduction

The origin and amplification of galactic and intergalactic magnetic fields constitutes one of the key open questions in contemporary astrophysics and cosmology [1–4]. Recent observations [5] of magnetic fields with strengths exceeding $B \sim 10^{-15}$ G in intergalactic space, including underdense regions (voids), has further enlivened the debate as to whether a dynamo-type mechanism could account for the presence of such fields, or whether some form of primordial magnetogenesis is required to explain them.

The present paper will mainly focus on the role of the magnetic field on very large spatial scales in a marginally open Friedmann universe. In [6, 7] (see also [4, 8–10]), it was argued that the substantially different evolution of the magnetic field in an open universe (as opposed to a universe which is either spatially flat or closed) may provide a mechanism by means of which a primordial magnetic field could survive until the present day, and remain astrophysically relevant. This idea is based on the dynamics of so-called supercurvature modes of the magnetic field in a spatially open expanding universe, whose decay rate with the cosmological scale factor a can be slower than the $B \propto a^{-2}$ decay, typical of a spatially flat universe.

The possibility of exciting supercurvature modes in an open universe remains to be clarified. Note that these modes do not belong to the space of square-integrable functions either on hypersurfaces of constant time in an open universe [11, 12], or on Cauchy hypersurfaces of the geodesically completed space-time [13, 14]. However, they have been used in [15] to describe perturbations with large correlation lengths in an open universe.

In this paper, we show that, even if supercurvature modes of electromagnetic field were present in a spatially open universe, their evolution would not help in solving the problem of primordial magnetogenesis in the manner suggested in [6, 7]. We demonstrate that the behaviour of magnetic fields in a marginally open universe (at the stage where $1 - \Omega(\eta) \ll 1$) is very similar to its behaviour in the spatially flat case, and that in both instances $B \propto a^{-2}$ with a high degree of accuracy. This result is shown to be valid during the exponential stage of open inflation as well as during the post-inflationary epoch, and holds for the magnetic

field on arbitrarily large spatial scales. In other words, a small value of the spatial curvature ensures that its effect on the magnetic field is tiny, and implies that a marginally open universe cannot either preserve or amplify a primordial magnetic field.

2   The magnetic field in an expanding universe

An expanding homogeneous and isotropic universe is described by the Friedmann-Robertson-Walker (FRW) metric which, in terms of the conformal time coordinate $\eta = \int cdt/a$, is written as

$$ds^2 = a^2(\eta)\left[d\eta^2 - \gamma_{ij}(x)dx^i dx^j\right]\,, \tag{2.1}$$

where $\gamma_{ij}(x)$ is the spatial homogeneous and isotropic metric, and c is the speed of light to be set to unity in what follows.

The components E_i and B_i of the observable electric and magnetic fields in such a universe are conveniently expressed through their conformal counterparts $\mathcal{E}_i$ and $\mathcal{B}_i$ as

$$B_i = \frac{1}{a^2}\mathcal{B}_i\,, \qquad E_i = \frac{1}{a^2}\mathcal{E}_i\,. \tag{2.2}$$

A free electromagnetic field obeys the Maxwell equations

$$\begin{aligned} \operatorname{div}\vec{\mathcal{B}} &= 0\,, \quad \operatorname{rot}\vec{\mathcal{E}} = -\vec{\mathcal{B}}'\,, \\ \operatorname{div}\vec{\mathcal{E}} &= 0\,, \quad \operatorname{rot}\vec{\mathcal{B}} = \vec{\mathcal{E}}'\,, \end{aligned} \tag{2.3}$$

where a prime denotes partial derivative with respect to the conformal time η, and the divergence and rotor operations are defined in the curved three-dimensional space with the metric γ_{ij}, associated covariant derivative ∇_i and normalized volume element ϵ_{ijk} as follows:

$$\operatorname{div}\vec{a} \equiv \nabla_i a^i\,, \qquad (\operatorname{rot}\vec{a})^i \equiv \epsilon^{ijk}\nabla_j a_k\,. \tag{2.4}$$

All spatial indices are raised and lowered here using the spatial metric γ_{ij} and its inverse.

From the Maxwell equations (2.3) one easily obtains a closed equation for the conformal magnetic field:

$$\mathcal{B}_i'' - \nabla^k\nabla_k\mathcal{B}_i + R_i{}^k\mathcal{B}_k = 0\,, \tag{2.5}$$

where R_{ij} is the Ricci tensor for the metric γ_{ij}. In the FRW case, we have $R_i{}^j = 2\kappa\delta_i{}^j$, so that the previous equation becomes

$$\mathcal{B}_i'' - \left(\nabla^k\nabla_k - 2\kappa\right)\mathcal{B}_i = 0\,. \tag{2.6}$$

Here, $\kappa = 0, \pm 1$ corresponds to the spatial curvature of the homogeneous and isotropic metric γ_{ij}. A similar equation is obtained for the conformal electric field $\mathcal{E}_i$.

By separation of time and space variables, the field is decomposed into transverse vector harmonics which are eigenfunctions of the Laplace operator $\nabla^k\nabla_k$ with eigenvalues[1] $-n^2$, and, for the coefficients $\mathcal{B}_{(n)}(\eta)$ of this decomposition, one obtains the ordinary differential equation

$$\mathcal{B}_{(n)}'' + \left(n^2 + 2\kappa\right)\mathcal{B}_{(n)} = 0\,. \tag{2.7}$$

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[1]In this paper, we follow the notation of [6, 7] for the eigenvalues of the Laplace operator for transverse vector fields.

Restricted to the space of square-integrable vector functions, the Laplace operator has a continuum spectrum with $n^2 > -2\kappa$ in the flat or open geometry, and a discrete spectrum with $n^2 = p^2 - 2$, $p = 2, 3, \ldots$, in the closed geometry (see, e.g., [16]). As can be seen from (2.7), all modes in this spectrum have harmonic oscillatory solutions

$$\mathcal{B}_{(n)} = C_1 \cos\left(\eta\sqrt{n^2 + 2\kappa}\right) + C_2 \sin\left(\eta\sqrt{n^2 + 2\kappa}\right), \tag{2.8}$$

where C_1 and C_2 are integration constants, so that the amplitudes of components (2.2) of the observable magnetic (and electric) field decay as $B \propto a^{-2}$.

In the case of a spatially open universe, $\kappa = -1$, the solutions of (2.7) with $n^2 < 2$ are usually called *supercurvature modes* (our definition of supercurvature modes matches that in [14], while only modes with $0 \le n < 1$ are referred to as being supercurvature in [6]). As functions of the radial coordinate, such modes do not oscillate but exhibit purely hyperbolic behaviour;[2] in this sense, they cannot be properly characterized by a wavelength. Although the supercurvature modes do not belong to the space of square integrable functions, their scalar counterparts were used to describe perturbations with large correlation lengths in a spatially open cosmological model [15].

As follows from (2.8), the supercurvature vector modes also have a qualitatively different temporal behaviour[3]

$$\mathcal{B}_{(n)} = C_1 e^{\eta\sqrt{2-n^2}} + C_2 e^{-\eta\sqrt{2-n^2}}. \tag{2.9}$$

The growing mode in (2.9) allows us to relate the final value of the magnetic field, $\mathcal{B}_{(n)}^{(\mathrm{f})}$, to its initial value, $\mathcal{B}_{(n)}^{(\mathrm{i})}$, as follows:

$$\frac{\mathcal{B}_{(n)}^{(\mathrm{f})}}{\mathcal{B}_{(n)}^{(\mathrm{i})}} = e^{\alpha(\eta_{\mathrm{f}} - \eta_{\mathrm{i}})} \equiv e^{\alpha\Delta\eta}, \tag{2.10}$$

where $\Delta\eta = \eta_{\mathrm{f}} - \eta_{\mathrm{i}}$ and $\alpha = \alpha(n) = \sqrt{2 - n^2}$. This equation implies that the temporal change in $\mathcal{B}_{(n)}$ depends upon the time span $\Delta\eta$. At this point, we should emphasize that, since $a(\eta)d\eta = dt$ determines the physical cosmological time, the scale of the conformal time is uniquely fixed by the normalization of the scale factor a. In a spatially closed or open universe, it is fixed by the conventional choice $\kappa = \pm 1$ for the spatial curvature of the metric γ_{ij} in (2.1). With this choice, both the scale factor a and the scale of the conformal time η in (2.1) have absolute geometrical, and therefore also physical, meaning.

Two extreme cases will be of interest to us in this paper: (a) $\Delta\eta \gg 1$, since this limit played a key role in the deductions of [6, 7]; (b) $\Delta\eta \ll 1$, which we show to provide a more accurate description of the phase of exponential inflation as well as the post-inflationary epoch. As one can see from (2.10), during epochs spanning a large range of values of the

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[2]The corresponding scalar radial mode characterized by the number n and by the angular number l has the behaviour [13, 14]

$$f_{nl}(r) \propto \sinh^l r \frac{d^l}{d\left(\cosh r\right)^l} \left(\frac{\sin\sqrt{n^2 - 2}\, r}{\sinh r}\right).$$

For $n^2 > 2$, the modes oscillate with comoving wavelength $\lambda = 2\pi/\sqrt{n^2 - 2}$. For $n^2 < 2$, they exhibit purely hyperbolic behaviour. Transverse vector modes are constructed from the scalar modes by additional differentiation [14].

[3]In [9], the difference between (2.8) and (2.9) was attributed to the fact that an open FRW universe is locally (but not globally) conformal to the flat Minkowski space [17].

conformal time, namely $\alpha\Delta\eta \gg 1$, the asymptotic growth of supercurvature modes of $\mathcal{B}_{(n)}$ is exponential:

$$\mathcal{B}_{(n)} \simeq \mathcal{B}_{(n)}^{(i)} e^{\alpha(\eta - \eta_i)} \tag{2.11}$$

whereas in the opposite case, when $\Delta\eta \ll 1$, the field $\mathcal{B}_{(n)}$ freezes at a constant value, $\mathcal{B}_{(n)}^{(f)} \simeq \mathcal{B}_{(n)}^{(i)}$, implying $B \propto a^{-2}$ even for supercurvature modes.

3 Large values of the conformal time

A homogeneous and isotropic universe with metric (2.1) is described by the Friedmann equation

$$\mathcal{H}^2 + \kappa = \frac{8\pi G}{3} a^2 \sum_i \rho_i \,, \tag{3.1}$$

together with the conservation equation for the density component ρ_i:

$$\rho_i' + 3\mathcal{H}(\rho_i + p_i) = 0 \,. \tag{3.2}$$

Here, $\mathcal{H} \equiv a'/a = aH$ is the conformal version of the Hubble parameter $H \equiv \dot{a}/a$, and an overdot denotes the derivative with respect to the physical time t. Of special significance will be the curvature parameter for such a universe:

$$\Omega_\kappa(\eta) = 1 - \Omega(\eta) \equiv -\frac{\kappa}{a^2 H^2} = -\frac{\kappa}{\mathcal{H}^2} \,. \tag{3.3}$$

An exact solution of (3.1), (3.2) for a spatially open universe ($\kappa = -1$) filled with matter with constant parameter of equation of state $w \equiv p/\rho$ is easily found to be (see also [6, 10])

$$a(\eta) = a_* \left(\frac{\sinh\beta\eta}{\sinh\beta\eta_*} \right)^{1/\beta} \,, \tag{3.4}$$

where

$$\beta = \frac{1 + 3w}{2} \neq 0 \,, \tag{3.5}$$

and η_* and $a_* = a(\eta_*)$ are integration constants. For the expansion law (3.4), one finds

$$\mathcal{H} = \coth\beta\eta \,, \tag{3.6}$$

and the curvature parameter (3.3) has the form

$$\Omega_\kappa(\eta) = \tanh^2\beta\eta \,, \tag{3.7}$$

so that a large absolute value of η in (3.7) implies a large value of $\Omega_\kappa(\eta)$, and, conversely, a small value of $\Omega_\kappa(\eta)$ implies a small absolute value for η.

Two toy models played a key role in the magnetic-field analysis of [6, 9], namely:

(i) $a \propto (\sinh\eta)^{-1}$, which describes open inflation and corresponds to $w = -1 \Rightarrow \beta = -1$ in (3.4)–(3.7).

(ii) $a \propto \sinh\eta$, which describes an open radiation-dominated universe and corresponds to $w = 1/3 \Rightarrow \beta = 1$ in (3.4)–(3.7).

The value of the conformal time η in (3.4) is calculated from the past cosmological singularity ($a = 0$) if $\beta > 0$, and from the future asymptotic infinity ($1/a = 0$) if $\beta < 0$. In both cases, for a large span of the conformal time parameter, namely, for $|\beta|\Delta\eta \gg 1$, the scale factor (3.4) evolves exponentially with η,

$$a(\eta) \propto e^{\eta}\,, \tag{3.8}$$

and describes the empty Milne universe. For (i) the Milne asymptote precedes exponential inflation (with $a(\eta) \propto -1/\eta \propto e^{Ht}$) and therefore lies in the remote past, whereas for (ii) the Milne asymptote succeeds a radiation-dominated epoch and therefore lies in the remote future.[4] The presence of the Milne asymptote is quite general for arbitrary β, or even for a spatially open universe filled with arbitrary matter, and arises whenever the total density of this matter is subdominant to the spatial curvature in (3.1), i.e., when

$$\frac{8\pi G}{3}a^2 \sum_i \rho_i \ll 1\,, \tag{3.9}$$

and $\Omega_\kappa(\eta) \simeq 1$ in (3.3).

During the curvature-dominated regime described by (3.8), the evolution (2.9) of the supercurvature modes of the magnetic field can be presented as

$$\mathcal{B}_{(n)} = C_1 a^{\sqrt{2-n^2}} + C_2 a^{-\sqrt{2-n^2}}\,, \tag{3.10}$$

so that

$$B_{(n)} = C_1 a^{\sqrt{2-n^2}-2} + C_2 a^{-\sqrt{2-n^2}-2}\,, \tag{3.11}$$

and the first mode of the observable magnetic field in (3.11) decays considerably less rapidly than given by the usual law $B \propto a^{-2}$. This effect was highlighted in a number of papers [4, 6–10] and led to the claim that it remained valid during "a period of slow-roll inflation, during reheating, and subsequently in the radiation and dust epochs," and that throughout this time large-scale B-fields were "*superadiabatically amplified*[5] by curvature effects alone" [7]. As we have seen, the Milne asymptote (3.8), so very vital for the derivation of (3.10) and (3.11), and the basis for the claim that supercurvature modes of the B-field are superadiabatically amplified, demands $\Omega_\kappa(\eta) \simeq 1$. As we proceed to show in the next section, the transition from eq. (2.9) to eqs. (3.10) and (3.11), which is precisely the transition from eq. (5) to eq. (6) in ref. [7], is valid only at a possible early stage of cosmological expansion dominated by spatial curvature ('coasting phase' in the terminology of [6]) but is erroneous at the subsequent stage of exponential inflation and also after inflation, during which $\Omega_\kappa(\eta) \ll 1$ and the magnetic field decays as $B \propto a^{-2}$.

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[4]Solution (3.4) played an important role in the conclusions drawn in [6, 7]. However, this framework is somewhat simplistic since the universe has several components, including dark energy. As we shall show in the next section, the asymptotic expansion law (3.8) characteristic of an empty Milne universe is an oversimplification which is never achieved in the real universe with the cosmological constant.

[5]The *superadiabatic amplification* of the electromagnetic field by curvature effects should be distinguished from the superadiabatic amplification of quantum fields which takes place during inflation. In the case of the latter, the amplitude of a given mode that leaves the Hubble radius during inflation is superadiabatically amplified relative to a much higher momentum mode which never left the Hubble radius and whose amplitude therefore decreased adiabatically throughout. Superadiabatic amplification in quantum language translates into particle production, and prominent examples of this process include the inflationary production of gravity waves [18] and fields that couple non-minimally to gravity [2, 19–21]. Note that this does not happen in the case of the electromagnetic field, which couples conformally to gravity and whose modes, therefore, remain in the vacuum state throughout the expansion of a FRW space-time [2, 11, 12].

4 Small values of the conformal time

The authors of [6–10] used expressions (3.10) and (3.11) during the inflationary as well as post-inflationary (radiation and matter dominated) stages to sustain magnetic fields on large spatial scales in supercurvature modes. However, as we have already noted, the asymptotic expressions (3.10) and (3.11) were obtained under the assumption of the exponential behaviour (3.8) of the scale factor, which is valid only when the conformal time parameter in (3.4) spans a large range of values. Unfortunately, this condition can be valid only in a curvature-dominated universe, and does not apply to the phase of exponential inflation and post-inflationary epoch, during which both $\Omega_\kappa(\eta)$ and $\Delta\eta$ are small. Let us consider this issue in more detail.

4.1 Magnetic-field evolution during open inflation

Cosmological inflation is usually driven by an ingredient (a scalar field) with effective equation of state $w \approx -1$. Setting $w = -1$ in (3.4) and (3.5) results in the following expansion law for open inflation:

$$a(\eta) = a_* \frac{\sinh \eta_*}{\sinh \eta} \,. \tag{4.1}$$

In this conventional expression, the conformal time η is counted from the future asymptotic infinity (where $1/a = 0$). It spans negative values and is decreasing by absolute magnitude as the universe expands. Since $\mathcal{H} = -\coth\eta$, the deceleration parameter in such a universe,

$$q \equiv -\frac{\ddot{a}}{aH^2} = -\frac{\mathcal{H}'}{\mathcal{H}^2} = -\frac{1}{\cosh^2 \eta} \,, \tag{4.2}$$

describes the 'coasting' (Milne) phase $a(t) \propto t$ when $|\eta| \gg 1$ with $|q| \ll 1$, and the phase of exponential inflation $a(t) \propto e^{Ht}$ when $|\eta| \ll 1$ (with $|q| \to 1$ as $\eta \to 0$). The fact that the absolute value of η is small during inflation can also be seen from the expression for the curvature parameter (3.7). Setting $\beta = -1$ in (3.7), we obtain

$$\Omega_\kappa(\eta) = \tanh^2 \eta \,, \tag{4.3}$$

which reflects the geometrical meaning of the value of the conformal time, as noted in the previous section. It is sensible to agree that inflation 'commences' when the inflaton energy density begins to dominate the spatial curvature. This occurs when the value of the curvature parameter has dropped below $\Omega_\kappa = 0.5$. From (4.3) we find the corresponding value of η to be $|\eta| \approx 0.88$. Hence, $|\eta| < 1$ during inflation.

Well within the regime of exponential inflation, one has $|\eta| \ll 1$, and (4.1) reduces to the usual flat-space behaviour

$$a(\eta) \approx a_* \frac{\sinh \eta_*}{\eta} \,. \tag{4.4}$$

Equation (2.9) in this case takes the form

$$\mathcal{B}_{(n)} \approx \tilde{C}_1 + \tilde{C}_2\, \eta\, \sqrt{2 - n^2} \simeq \tilde{C}_1 \,, \tag{4.5}$$

where $\tilde{C}_1 = C_1 + C_2$ and $\tilde{C}_2 = C_1 - C_2$. In other words, since the span of conformal time during exponential inflation is small ($\Delta\eta \lesssim 1$), the field $\mathcal{B}_{(n)}$ tends to a constant value and the physical magnetic field asymptotically decays as $B \propto a^{-2}$. This leads us to conclude that no 'superadiabatic amplification' of the magnetic field occurs during exponential inflation.

The previous reasoning was based on the de Sitter solution (4.1). However, a generic estimate can be made for the span $\Delta\eta$ of the conformal time in any model of open quasi-exponential inflation, satisfying the condition $|\dot{H}| \ll H^2$, between the moments of its beginning and its end:

$$\Delta\eta = \int_{t_{\rm in}}^{t_{\rm end}} \frac{dt}{a(t)} = \int_{a_{\rm in}}^{a_{\rm end}} \frac{da}{a^2 H} \simeq \frac{1}{a_{\rm in} H_{\rm in}} = \sqrt{\Omega_\kappa^{\rm in}}\,. \tag{4.6}$$

Since, in any case, $\Omega_\kappa^{\rm in} < 1$, we get an upper bound $\Delta\eta \lesssim 1$.

It is instructive to estimate the value of Ω_κ at the end of inflation (or at the commencement of reheating, which, for simplicity, is assumed to start immediately after the end of inflation):

$$\Omega_\kappa(\eta_{\rm end}) \simeq \Omega_\kappa(\eta_{\rm rh}) \simeq \left(\frac{g_0}{g_{\rm rh}}\right)^{1/3} \left(\frac{T_0}{T_{\rm rh}}\right)^2 \frac{\Omega_{0\kappa}}{\Omega_{0\gamma}} \simeq 10^{-52} \left(\frac{10^3}{g_{\rm rh}}\right)^{1/3} \left(\frac{10^{15}\,{\rm GeV}}{T_{\rm rh}}\right)^2 \Omega_{0\kappa}\,. \tag{4.7}$$

Here, $T_{\rm rh}$ is the temperature of reheating, $T_0 \approx 2.7$ K $\simeq 2.3 \times 10^{-4}$ eV is the current temperature of the cosmic microwave background (CMB), while $\Omega_{0\gamma} \approx 5 \times 10^{-5}$ is the current value of the CMB energy density. The quantity $g_0 = 2$ is the number of degrees of freedom of the photon and $g_{\rm rh}$ is the number of relativistic degrees of freedom in thermal equilibrium after reheating. The current value of the curvature parameter in the ΛCDM model is constrained by observations [22] to lie in the interval $-0.0133 < \Omega_{0\kappa} < 0.0084$ (95% CL), so it is conceivable that $\Omega_{0\kappa} \simeq 10^{-2}$, which was suggested in [6, 7] to describe a marginally open universe. Substituting $\Omega_{0\kappa} = 10^{-2}$ in (4.7), we obtain $\Omega_\kappa(\eta_{\rm end}) \approx \eta_{\rm end}^2 \simeq 10^{-54}$ at the end of inflation. Consequently, $|\eta| \simeq 10^{-27}$ when inflation ended, and $|\eta| \sim 1$ when exponential inflation commenced (about 62 e-foldings before the end of inflation). Thus, $|\eta|$ remains small during the entire duration of exponential inflation, and we arrive at the conclusion that (4.5), rather than (3.10), provides the correct description of the behaviour of the magnetic field during inflation in an open universe.

4.2 The magnetic field in the post-inflationary universe

As previously noted, a key role in the derivation of superadiabatic amplification of large-scale B-modes in (3.10) and (3.11) was played by the assumption that the cosmological scale factor displays behaviour characteristic of the curvature-dominated Milne universe, namely $a \propto e^\eta$. However, as we have seen in a previous subsection, the behaviour of the scale factor during exponential inflation is $a(\eta) \propto -1/\eta$ with $\eta \to 0^-$, so that no superadiabatic amplification occurs at this stage.

In [9], it has been claimed that the superadiabatic amplification of the B field is not confined to the inflationary phase but extends also to the radiation and matter dominated epochs which follow preheating. For this to be the case, the expansion law $a \propto e^\eta$ needs to be valid during radiation and matter domination. In this section, we show that this is not the case and that the Milne asymptote $a \propto e^\eta$, characteristic of a curvature-dominated empty universe, is never followed during post-inflationary expansion. The reason of this behaviour is quite similar to that discussed in the previous subsection in connection with exponential inflation: during post-inflationary expansion, the physically relevant conformal time η spans a small range of values $\Delta\eta \ll 1$, commencing from exceedingly small initial values and remaining small until today. As a consequence, the scale factor $a(\eta)$ behaves as a power of η, rather then as an exponent.

A spatially open universe filled with radiation and matter is described by the following exact solution of the Friedmann equation (3.1) [23]:

$$a(\eta) = a_{\mathrm{eq}} \left(\zeta \sinh \eta + \zeta^2 \sinh^2 \frac{\eta}{2} \right) , \qquad \eta > 0 , \tag{4.8}$$

where

$$\zeta = \left(\frac{8\pi G}{3} \rho_{\mathrm{eq}} a_{\mathrm{eq}}^2 \right)^{1/2} = \left(\frac{1 - \Omega_{\kappa\,\mathrm{eq}}}{2\Omega_{\kappa\,\mathrm{eq}}} \right)^{1/2} = \left[\frac{\Omega_{\mathrm{eq}}}{2\left(1 - \Omega_{\mathrm{eq}}\right)} \right]^{1/2} , \tag{4.9}$$

the subscript 'eq' refers to the moment of equality of matter and radiation densities, and ρ_{eq} is the density of matter or radiation at that moment. The conformal time at this stage of expansion is conventionally counted from the extrapolated cosmological singularity ($a = 0$ as $\eta = 0$).

The curvature parameter (3.3) for solution (4.8) is given by[6]

$$\Omega_{\kappa}(\eta) = \left(\frac{\sinh \eta + \zeta \sinh^2 \frac{\eta}{2}}{\cosh \eta + \zeta \cosh \frac{\eta}{2} \sinh \frac{\eta}{2}} \right)^2 , \tag{4.10}$$

from which we find that the exceedingly small value of $\Omega_{\kappa}(\eta)$ during the radiation and matter dominated epochs implies $\eta \ll 1$. (In fact, by assuming, for simplicity, instantaneous reheating and matching the parameter Ω_{κ} at the end of inflation and at reheating, we immediately get the relation $\eta_{\mathrm{rh}} \simeq |\eta_{\mathrm{end}}|$ between the values of the conformal times at reheating and at the end of inflation.) Such a small value of η ensures that equation (4.8) reduces to its flat-space counterpart

$$a(\eta) \approx a_{\mathrm{eq}} \left(\zeta\eta + \frac{1}{4}\zeta^2\eta^2 \right) , \tag{4.11}$$

and the evolution (2.9) of the conformal magnetic field is approximated as

$$\mathcal{B}_{(n)} \approx \tilde{C}_1 + \tilde{C}_2 \sqrt{2 - n^2}\, \eta \simeq \tilde{C}_1 , \tag{4.12}$$

implying $B \propto a^{-2}$. We, therefore, conclude that the large-η asymptote $a \propto e^{\eta}$ used in the derivation of superadiabatic amplification in (3.10) is inapplicable, and the more relevant equations (4.11) and (4.12) imply that the magnetic field decreases in much the same manner as it does in a spatially flat universe, namely $B \propto a^{-2}$.

Equation (4.8), which takes into account the effect of spatial curvature but misses the effect of dark energy, accurately describes the real universe only until dark energy starts dominating over the spatial curvature. In the ΛCDM model, this takes place at redshift $z = \sqrt{\Omega_{\Lambda}/\Omega_{0\kappa}} - 1 \approx 7.5$ for $\Omega_{\Lambda} = 0.73$ and $\Omega_{0\kappa} = 0.01$. After the cosmological constant starts dominating absolutely, we proceed to an asymptotically de Sitter stage again. Therefore, during the post-inflationary expansion of a ΛCDM universe, the curvature term is always strongly subdominant. Consequently, the Milne expansion law $a \propto e^{\eta}$, which was crucial in the transition from eq. (2.9) to eqs. (3.10) and (3.11) made in [6–10], is never (even asymptotically) valid in a ΛCDM universe.

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[6]Only the first term in the brackets of (4.8) was retained in [9] for a description of a radiation-dominated universe, which is somewhat misleading because matter plays a singularly more important role than spatial curvature during this epoch. Indeed, taking the cosmological matter parameter $\Omega_{0m} \simeq 1/3$ and $\Omega_{0\kappa} \simeq 10^{-2}$, we have $\frac{\Omega_{\kappa}(z)}{\Omega_m(z)} = \frac{\Omega_{0\kappa}}{\Omega_{0m}}(1+z)^{-1}$, so that the ratio of curvature to matter becomes a minuscule 10^{-5} at $z \sim 10^4$, and was even smaller earlier on.

Since the numerical value of the conformal time has played a key role in the above discussion, we give an exact formula for the span of η in a spatially open post-inflationary Friedmann universe. The Hubble parameter has the form

$$h(z) \equiv \frac{H(z)}{H_0} = \left[\Omega_{0r}(1+z)^4 + \Omega_{0m}(1+z)^3 + \Omega_{0\kappa}(1+z)^2 + \Omega_\Lambda \right]^{1/2} , \qquad (4.13)$$

where we assume for simplicity that the role of dark energy is played by the cosmological constant and take into account that the contribution to pressureless matter comes from baryons as well as dark matter. The value of the curvature parameter at an earlier epoch is related to its present value as

$$\Omega_k(z) = \Omega_{0\kappa} \frac{(1+z)^2}{h^2(z)} , \qquad (4.14)$$

while the conformal time coordinate as a function of redshift has the form

$$\eta(z) = \frac{1}{a_0 H_0} \int_z^\infty \frac{dx}{h(x)} , \qquad (4.15)$$

which reduces to

$$\eta(z) = \sqrt{\Omega_{0\kappa}} \int_z^\infty \frac{dx}{h(x)} \qquad (4.16)$$

in a spatially open universe.

A simple calculation shows that the value of η was small in the past and shall remain small also in the future. Note that, for $z \geq 0$, the integral in (4.16) is bounded by its present value:

$$\int_z^\infty \frac{dz}{h(z)} \leq \int_0^\infty \frac{dz}{h(z)} \simeq 3.4 . \qquad (4.17)$$

Substituting $\Omega_{0\kappa} = 10^{-2}$ in (4.16), therefore, implies $\eta(z) < 0.34$ for $z > 0$. The total span of the conformal time in the ΛCDM model is obtained by integrating from $z = -1$ in (4.16), with the result $\Delta\eta = \eta(-1) \approx 0.45$ for $\Omega_{0\kappa} = 10^{-2}$. These results are borne out by an exact determination of $\eta(z)$ whose values are shown in table 1 for different cosmological redshifts, assuming present-day cosmological parameters which are consistent with the data analysis of the Wilkinson Microwave Anisotropy Probe [22].

z	$\Omega_\kappa(z)$	$\eta(z)$
-1	0	0.45
0	0.01	0.34
10	3.5×10^{-3}	0.11
3200	7.4×10^{-6}	2.7×10^{-3}
10^9	2×10^{-16}	1.4×10^{-8}

Table 1. The spatial-curvature parameter $\Omega_\kappa(z)$ in (4.14) and the conformal time coordinate $\eta(z)$ in (4.16) are shown for typical values of the cosmological redshift.

5   The magnetic field on large spatial scales

In the previous section, we have demonstrated the absence of superadiabatic amplification of the magnetic field at the stage of exponential inflation and after inflation. In view of these results, we would like to revise the estimates of [6, 7] for possible current values of the magnetic field. Considering the modes with $1 < n^2 < 2$, the authors of [6, 7] specified their initial amplitudes at the respective moments of time defined by the condition $n^2 = \mathcal{H}^2(\eta_{\mathrm{HC}})$ and termed 'horizon crossing.' Since the vector modes under consideration do not exhibit oscillatory behaviour either in the radial coordinate or in time, and, therefore, cannot be characterized by a real wavelength or frequency (see the discussion in section 2 and footnote 2), this characterization of 'horizon crossing' appears somewhat artificial. Nevertheless, following [6, 7], we assume these initial values for the magnetic-field amplitudes and trace their evolution in an open universe. Even in this case, as we are going to show, our results turn out to be quite different from those derived in [6, 7].

During the epoch of open inflation, described by (4.1), the condition of 'horizon crossing' $n^2 = \mathcal{H}^2(\eta_{\mathrm{HC}})$ translates into

$$n^2 = \coth^2 \eta_{\mathrm{HC}} \quad \Rightarrow \quad \eta_{\mathrm{HC}} = -\coth^{-1} n \tag{5.1}$$

(remember that $\eta < 0$ at this stage). Note that the relation (3.3) implies

$$\Omega_k(\eta_{\mathrm{HC}}) = \frac{1}{n^2}, \tag{5.2}$$

which informs us that the moment of 'horizon crossing' for modes with lower n occurs when the value of Ω_k is larger (see also [6]).

The magnetic-field modes under consideration evolve according to (2.9). Since our aim is to see what maximal possible magnetic field one can obtain today, we retain only the growing mode in this equation. At the initial moment of 'horizon crossing,' we then have

$$\mathcal{B}_{\mathrm{HC}} = C_1 e^{\eta_{\mathrm{HC}}\sqrt{2-n^2}} = C_1 \left(\frac{n-1}{n+1} \right)^{\frac{\sqrt{2-n^2}}{2}}. \tag{5.3}$$

Towards the end of inflation, we have $|\eta_{\mathrm{end}}| \lesssim 10^{-27} \ll 1$ (see section 4.1), which leads to

$$\mathcal{B}_{\mathrm{end}} \simeq C_1 \approx \left(\frac{n+1}{n-1} \right)^{\frac{\sqrt{2-n^2}}{2}} \mathcal{B}_{\mathrm{HC}}. \tag{5.4}$$

Since $B = \mathcal{B}/a^2$, one obtains the following relation for physical magnetic fields:

$$B_{\mathrm{end}} \simeq \left(\frac{a_{\mathrm{HC}}}{a_{\mathrm{end}}} \right)^2 \left(\frac{n+1}{n-1} \right)^{\frac{\sqrt{2-n^2}}{2}} B_{\mathrm{HC}}. \tag{5.5}$$

As we have shown in section 4.2, during post-inflationary era, the magnetic field in all modes decays approximately as $B \propto a^{-2}$. Consequently, for the present value of the magnetic field, one gets

$$B_0 = \left(\frac{a_{\mathrm{end}}}{a_0} \right)^2 B_{\mathrm{end}} \approx \left(\frac{a_{\mathrm{HC}}}{a_0} \right)^2 \left(\frac{n+1}{n-1} \right)^{\frac{\sqrt{2-n^2}}{2}} B_{\mathrm{HC}}. \tag{5.6}$$

The ratio of the scale factors in (5.6) is determined using (4.3) and (5.1), and, by keeping in mind the extreme smallness of $|\eta_{\text{end}}|$, we have

$$\frac{a_{\text{HC}}}{a_{\text{end}}} = \frac{\sinh \eta_{\text{end}}}{\sinh \eta_{\text{HC}}} = -\sqrt{n^2 - 1}\, \sinh \eta_{\text{end}} \approx \sqrt{(n^2 - 1)\Omega_\kappa^{\text{end}}}\,, \tag{5.7}$$

$$\frac{a_{\text{end}}}{a_0} = \frac{a_{\text{end}} H_{\text{end}}}{a_0 H_0} \cdot \frac{H_0}{H_{\text{end}}} = \sqrt{\frac{\Omega_{0\kappa}}{\Omega_\kappa^{\text{end}}}} \cdot \frac{H_0}{H_{\text{end}}}\,, \tag{5.8}$$

so that

$$\frac{a_{\text{HC}}}{a_0} = \frac{a_{\text{HC}}}{a_{\text{end}}} \cdot \frac{a_{\text{end}}}{a_0} = \sqrt{(n^2 - 1)\Omega_{0\kappa}}\, \frac{H_0}{H_{\text{end}}}\,. \tag{5.9}$$

Substituting this into (5.6), we get

$$B_0 \simeq (n+1)^{\sqrt{2-n^2}} \left(n^2 - 1\right)^{\frac{2-\sqrt{2-n^2}}{2}} \Omega_{0\kappa} \frac{H_0^2}{H_{\text{end}}^2} B_{\text{HC}}\,. \tag{5.10}$$

If, following [6, 8], we assume that $B_{\text{HC}} \simeq H_{\text{end}}^2$, then, from (5.10), we obtain[7]

$$\begin{aligned} B_0 &\simeq (n+1)^{\sqrt{2-n^2}} \left(n^2 - 1\right)^{\frac{2-\sqrt{2-n^2}}{2}} \Omega_{0\kappa} H_0^2 \\ &\simeq 10^{-63}(n+1)^{\sqrt{2-n^2}} \left(n^2 - 1\right)^{\frac{2-\sqrt{2-n^2}}{2}} \Omega_{0\kappa} h^2\, \text{G}\,, \end{aligned} \tag{5.11}$$

where $h = H_0 / \left(100 \text{ km s}^{-1} \text{ Mpc}^{-1}\right)$. Remarkably, the final value of the magnetic field in (5.11) does not depend upon the energy scale of inflation. Equation (5.11) should be compared with formula (35) of [6]:

$$B_0 \sim 10^{-65 + 51\sqrt{2-n^2}} \left(\frac{M}{10^{14} \text{ GeV}}\right)^{2\sqrt{2-n^2}} \left[(n^2 - 1)\Omega_{0\kappa}\right]^{\frac{2-\sqrt{2-n^2}}{2}} \text{ G}\,, \tag{5.12}$$

where M is the energy scale of inflation. The estimate for B_0 in (5.12) differs by many orders of magnitude from the correct estimate in (5.11) mainly because the authors of [6] employed the asymptotic formulas (3.10) and (3.11) not only at the coasting phase, where they are valid, but also at the inflationary stage and during post-inflationary evolution, where they are, in fact, inapplicable.

Our result (5.10) gives a very small estimate (5.11) for the magnetic field on supercurvature scales today. Even by increasing the energy density stored in the magnetic field during 'horizon crossing' to the inflationary energy density, we get $B_{\text{HC}} \sim M_{\text{P}} H_{\text{end}} \sim 10^5 H_{\text{end}}^2$ (here, M_{P} is the Planck mass), which, while increasing our estimate (5.11) by five orders of magnitude, still leaves it very small.

6 A bound on the free magnetic field in a marginally open universe

It is widely believed that, since Maxwell's equations couple conformally to gravity, a primordial magnetic field cannot be created solely by the expansion of a FRW universe, but might

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[7]Note that the assumption $B_{\text{HC}} \simeq H_{\text{end}}^2$ made in [6, 8] is somewhat speculative since hard calculations are missing in support of this point of view. Moreover, Adamek et al. [14] have recently drawn attention to an inherent difficulty in exciting supercurvature modes in open inflation and in the Milne universe.

do so if conformal invariance were somehow broken. Indeed, several attempts at magnetogenesis have introduced explicit couplings of electromagnetism either to gravity [19], or to the inflaton [24], to break conformal invariance and generate primordial magnetic fields.

As this paper was not directly concerned with the problem of magnetogenesis, we did not feel it appropriate to discuss this issue. Rather, since we were mainly concerned with revisiting some of the issues raised in [6, 8], we simply assumed, as did the authors of [6, 8], that the seed value of the magnetic field was linked to the Hubble value during inflation. One may, however, broaden the scope of the above discussion by asking whether an upper bound can be placed on the value of a non-interacting (free) primordial magnetic field by requiring that it remain compatible with open inflation.

Let us assume, for instance, that an open universe was created as a quantum bubble with some prevailing (seed) magnetic field. Initially, the energy density in such a universe may be dominated either by spatial curvature, or by the magnetic field, or by some other form of matter. We assume, however, that the universe at some moment of time $\eta_{\rm in}$ begins to inflate. After this moment, its Hubble expansion is dominated by the energy density of the inflaton, hence, both the magnetic-field energy density and the spatial curvature are subdominant. Such a universe quickly approaches the regime where it is adequately described by the FRW solution (4.1).

As we have shown in section 4, all modes of the free magnetic field decay approximately as $B \propto a^{-2}$ both during and after inflation. Since the magnetic field density was subdominant to that of the inflaton at the start of inflation, and since the free magnetic field does not experience any *additional* amplification (also true for the supercurvature modes, as discussed in section 4), we obtain a simple relation between the energy density contained in these modes at the beginning and at the end of exponential inflation:

$$\rho_{\rm end}^B \simeq \left(\frac{a_{\rm in}}{a_{\rm end}} \right)^4 \rho_{\rm in}^B = e^{-4N} \rho_{\rm in}^B < e^{-4N} \rho_{\rm in} \,, \tag{6.1}$$

where $\rho_{\rm in}$ is the initial energy density of the inflaton and N is the number of e-foldings during exponential inflation. For the current value of the magnetic field, we get

$$B_0 \sim \left(\frac{a_{\rm end}}{a_0} \right)^2 B_{\rm end} \sim \left(\frac{a_{\rm end}}{a_0} \right)^2 e^{-2N} \sqrt{\rho_{\rm in}^B} < \left(\frac{a_{\rm end}}{a_0} \right)^2 e^{-2N} \sqrt{\rho_{\rm in}}$$

$$\sim g_{\rm rh}^{-1/6} \left(\frac{\rho_{\rm in}}{\rho_{\rm end}} \right)^{1/2} e^{-2N} T_0^2 \simeq 2.5 \times 10^{-7} \left(\frac{10^3}{g_{\rm rh}} \right)^{1/6} \left(\frac{\rho_{\rm in}}{\rho_{\rm end}} \right)^{1/2} e^{-2N} \ {\rm G} \,. \tag{6.2}$$

The lower bound $N > 60$ on the number of e-foldings in the simplest inflationary models leads to the following upper bound on the present value of the magnetic field on supercurvature scales: $B_0 \lesssim 10^{-59}$ G. A significantly larger magnetic field, say, $B_0 \sim 10^{-16}$ G, would imply $N < 11$ and run into trouble with our current understanding of inflation.

7 Discussion

In this paper, we have investigated the evolution of supercurvature modes of the magnetic field in a marginally open universe. We have shown that, contrary to the claims made in [6, 7], such modes do not experience any significant amplification either during exponential inflation or during the post-inflationary epoch. The basic reason for this is that, while the conformal field $\mathcal{B} = a^2 B$ in these modes does evolve exponentially with conformal time, the

conformal time itself during these stages spans a very small range of values, $\Delta\eta \ll 1$, (while the scale factor $a(\eta)$ evolves as a power of η and not as $a \propto e^\eta$). Thus, the conformal magnetic field $\mathcal{B}$ remains effectively frozen in time, and the physical magnetic field B decays as a^{-2}. Following the evolution of the magnetic field in open inflation with initial conditions as in [6, 8], we arrive at a very small estimate (5.11) for its current value: $B_0 \lesssim 10^{-65}$ G.

By considering the contribution of the magnetic field to the rate of expansion of the universe, one can obtain a general upper bound on the residual free magnetic field in an open universe, compatible with a sufficiently long epoch of exponential inflation. For $N = 60$ inflationary e-foldings, we arrive at the estimate $B_0 \lesssim 10^{-59}$ G.

The possibility of exciting supercurvature modes of the magnetic field in an open inflationary universe was put into question in [14] basically because these modes do not belong to the space of square-integrable functions [13, 14]. In this paper, the important issue of excitation of supercurvature modes of the electromagnetic field has been set aside. Instead, we have shown that even if these modes were somehow present, their current amplitude in a marginally open universe would be too small to account for primordial magnetogenesis.

Acknowledgments

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